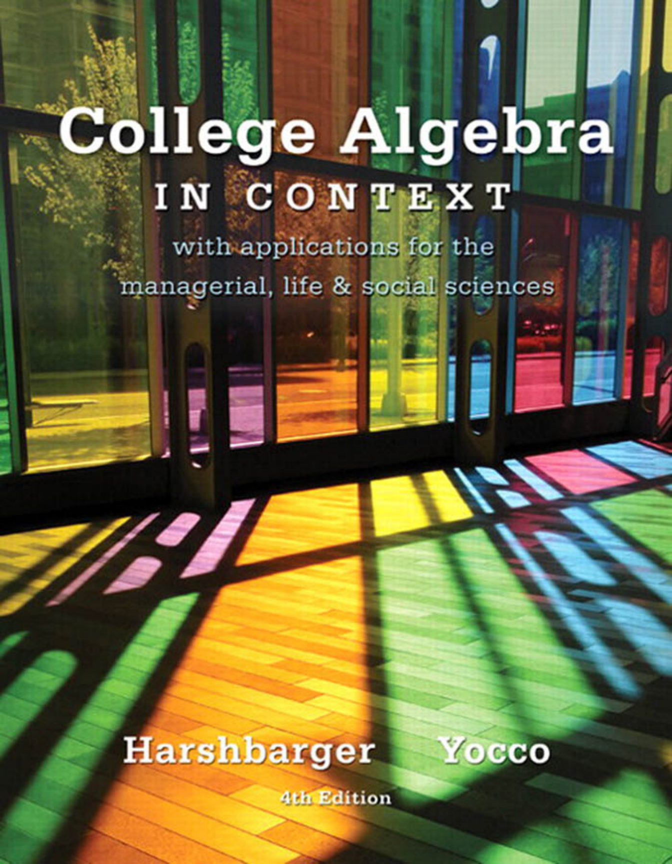

College Algebra
IN CONTEXT
with applications for the managerial, life & social sciences

Harshbarger Yocco

4th Edition

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with applications for the managerial, life, and social sciences

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Preface

College Algebra in Context is designed for a course in algebra that is based on data analysis, modeling, and real-life applications from the management, life, and social sciences. The text is intended to show students how to analyze, solve, and interpret problems in this course, in future courses, and in future careers. At the heart of this text is its emphasis on problem solving in meaningful contexts.

The text is application-driven and uses real-data problems that motivate interest in the skills and concepts of algebra. Modeling is introduced early, in the discussion of linear functions and in the discussion of quadratic and power functions. Additional models are introduced as exponential, logarithmic, logistic, cubic, and quartic functions are discussed. Mathematical concepts are introduced informally with an emphasis on applications. Each chapter contains real-data problems and extended application projects that can be solved by students working collaboratively.

The text features a constructive chapter-opening Algebra Toolbox, which reviews previously learned algebra concepts by presenting the prerequisite skills needed for successful completion of the chapter. In addition, a section on calculus preparation at the end of the text emphasizes how students can use their new knowledge in a variety of calculus courses.

Changes in the Fourth Edition

We have made a number of changes based on suggestions from users and reviewers of the third edition, as well as our own classroom experiences.

Chapter objectives are listed at the beginning of each chapter, and key objectives are given at the beginning of each section.

Appendix B has been expanded to include instructions for Excel 2007 and Excel 2010 as well as Excel 2003. Where a new technology procedure is introduced in the text, references to calculator and Excel instructions in Appendixes A and B have been added.

To keep the applications current, nearly all of the examples and exercises using real data have been updated or replaced. This is especially important in light of the numerous economic and financial changes that have occurred since publication of the previous edition.

Many new Section Previews and other applications provide motivation. Applications are referenced on the chapter-opening page in the order in which they will appear in the section.

To improve the exposition, the organization and content of some sections have been changed.

- Section 1.1 contains an expanded discussion of mathematical models.
- Section 1.3 provides a more complete introduction to revenue, cost, and profit functions.
- Section 2.1 includes expanded discussion of direct variation.
- The term "Power Functions" has been included in the Chapter 3 title to highlight the increased emphasis on this topic in this chapter.
- Section 3.1 contains an expanded discussion of graphing quadratic functions.
- Direct variation as an nth power and inverse variation have been added to Section 3.3.
- Additional discussion of graphing and solving equations with Excel has been added to Chapter 3.
- Chapter 4 Toolbox has been revised to contain a library of functions.

- A more complete introduction to the average cost function now appears in Section 4.2.
- The discussion of one-to-one functions has been moved to Section 4.3 with inverse functions.
- Section 5.1 has been reorganized to complete the discussion of exponential functions before moving on to exponential growth and decay.
- Additional Skills Check problems have been added to ensure that every type of skill is well represented.
- Additional modeling questions involving more decision making and critical thinking have been added throughout the text.
- Sample homework exercises, chosen by the authors, are indicated with an underline in the Annotated Instructor's Edition.

Continued Features

Features of the text include the following:

- The development of algebra is motivated by the need to use algebra to find the solutions to **real data–based applications**.

  Real-life problems demonstrate the need for specific algebraic concepts and techniques. Each section begins with a motivational problem that presents a real-life setting. The problem is solved after the necessary skills have been presented in that section. The aim is to prepare students to solve problems of all types by first introducing them to various functions and then encouraging them to take advantage of available technology. Special business and finance models are included to demonstrate the applications of functions to the business world.

- **Technology** has been integrated into the text.

  The text discusses the use of graphing calculators and computers, but there are no specific technology requirements. When a new calculator or spreadsheet skill becomes useful in a section, students can find the required keystrokes or commands in the *Graphing Calculator and Excel® Manual* that accompanies the text, as well as in the appendixes mentioned above. The text indicates where calculators and spreadsheets can be used to solve problems. Technology is used to enhance and support learning when appropriate—not to supplant learning.

- The text contains two technology appendixes: a **Basic Calculator Guide** and a **Basic Guide to Excel 2003, Excel 2007, and Excel 2010.** Footnotes throughout the text refer students to these guides for a detailed exposition when a new use of technology is introduced. Additional Excel solution procedures have been added, but, as before, they can be omitted without loss of continuity in the text.

- Each of the first seven chapters begins with an **Algebra Toolbox** section that provides the prerequisite skills needed for the successful completion of the chapter.

  Topics discussed in the Toolbox are topics that are prerequisite to a college algebra course (usually found in a Chapter R or appendix of a college algebra text). Key objectives are listed at the beginning of each Toolbox, and topics are introduced "just in time" to be used in the chapter under consideration.

- Many problems posed in the text are **multi-part and multi-level problems**.

  Many problems require thoughtful, real-world answers adapted to varying conditions, rather than numerical answers. Questions such as "When will this model no longer be valid?" "What additional limitations must be placed on your answer?" and "Interpret your answer in the context of the application" are commonplace in the text.

- Each chapter has a **Chapter Summary**, a **Chapter Skills Check**, and a **Chapter Review**.

  The Chapter Summary lists the key terms and formulas discussed in the chapter, with section references. Chapter Skills Check and Chapter Review exercises provide additional review problems.

- The text encourages **collaborative learning**.

  Each chapter ends with one or more Group Activities/Extended Applications that require students to solve multilevel problems involving real data or situations, making it desirable for students to collaborate in their solutions. These activities provide opportunities for students to work together to solve real problems that involve the use of technology and frequently require modeling.

- The text ends with a **Preparing for Calculus** section that shows how algebra skills from the first four chapters are used in a calculus context.

  Many students have difficulty in calculus because they have trouble applying algebra skills to calculus. This section reviews earlier topics and shows how they apply to the development and application of calculus.

- The text encourages students to improve **communication skills and research skills**.

  The Group Activities/Extended Applications require written reports and frequently require use of the internet or library. Some Extended Applications call for students to use literature or the internet to find a graph or table of discrete data describing an issue. They are then required to make a scatter plot of the data, determine the function type that is the best fit for the data, create the model, discuss how well the model fits the data, and discuss how it can be used to analyze the issue.

- **Answers to Selected Exercises** include answers to all Chapter Skills Checks and Chapter Reviews, so students have feedback regarding the exercises they work.

- **Supplements** are provided that will help students and instructors use technology to improve the learning and teaching experience. See the supplements list.

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Ronald J. Harshbarger
Lisa S. Yocco

* Denotes reviewers of the fourth edition.

List of Supplements

Student Supplements

Student's Solutions Manual

- By Lee Graubner, *Valencia Community College*.
- Provides selected solutions to the Skills Check problems and Exercises, as well as solutions to all Chapter Skills Checks, Review problems, Algebra Toolbox problems, and Extended Applications.
- ISBN-13: 978-0-321-78355-4; ISBN-10: 0-321-78355-7

A Review of Algebra

- By Heidi Howard, *Florida Community College at Jacksonville*.
- Provides additional support for those students needing further algebra review.
- ISBN-13: 978-0-201-77347-7; ISBN-10: 0-201-77347-3

Instructor Supplements

Annotated Instructor's Edition

- Provides answers to many text exercises right after the exercise and answers to all the exercises in the back of the book.
- Notes with an underline sample homework exercises, selected by the authors and supported by MathXL.
- ISBN-13: 978-0-321-78357-8; ISBN-10: 0-321-78357-3

Instructor's Solutions Manual

- By Lee Graubner, *Valencia Community College*.
- Provides complete solutions to all Algebra Toolbox problems, Skills Check problems, Exercises, Chapter Skills Checks, Review problems, and Extended Applications.
- ISBN-13: 978-0-321-78356-1; ISBN-10: 0-321-78356-5

Instructor's Testing Manual

- By Melanie Fulton.
- Contains three alternative forms of tests per chapter.
- Includes answer keys with more applications.
- Available for download from Pearson Education's online catalog.

TestGen®

- Enables instructors to build, edit, print, and administer tests.
- Features a computerized bank of questions developed to cover all text objectives.
- Algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button.
- Available for download from the Instructor Resource Center at pearsonhighered.com/irc.

Technology Resources

MathXL® Online Course (access code required)

MathXL is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning management system.) With MathXL, instructors can

- Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- Create and assign their own online exercises and import TestGen tests for added flexibility.
- Maintain records of all student work tracked in MathXL's online gradebook.

With MathXL, students can

- Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.
- Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.
- Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com or contact your Pearson representative.

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from a **trusted partner** with educational expertise and an eye on the future.

- **Narrated Example Videos with subtitles** have been updated to reflect new content and current real data. Available to download from within MyMathLab®, the videos are correlated to the examples in the Graphing Calculator and Excel® Manual and walk students through algebraic solutions to pivotal examples and provide technological solutions where applicable.
- **NEW! Introductory Videos** contain a brief overview of the topics covered in each section and highlight objectives and key concepts. These videos give context to the Narrated Example Videos and are available in MyMathLab®.
- **NEW! Interactive Figures** enable you to manipulate figures to bring math concepts to life. They are assignable in MyMathLab and available in the Multimedia Library.
- **NEW! Animations** have been added to the Algebra Toolbox section to help students master the prerequisite skills needed to be successful in the chapter. Instructors can assign these multimedia learning aids as homework to help their students grasp the concepts.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit www.mymathlab.com or contact your Pearson representative.

MyMathLab® Ready to Go Course (access code required)

These new Ready to Go courses provide students with all the same great MyMathLab features that you're used to, but make it easier for instructors to get started. Each course includes preassigned homeworks and quizzes to make creating your course even simpler. Ask your Pearson representative about the details for this particular course or to see a copy of this course.

To the Student

College Algebra in Context was written to help you develop the math skills you need to model problems and analyze data—tasks that are required in many jobs in the fields of management, life science, and social science. As you read this text, you may be surprised by how many ways professionals use algebra—from predicting how a population will vote in an upcoming election to projecting sales of a particular good.

There are many ways in which this text will help you succeed in your algebra course, and you can be a partner in that success. Consider the following suggestions, which our own students have found helpful.

1. **Take careful notes in an organized notebook.** Good organization is essential in a math course so that you do not fall behind and so that you can quickly refer back to a topic when you need it.
   - **Separate your notebook into three sections:** class notes and examples, homework, and a problem log consisting of problems worked in class (which will provide a sample test).
   - **Begin each set of notes with identifying information:** the date, section of the book, page number from the book, and topic.
   - **Write explanations in words**, rather than just the steps to a problem, so that you will understand later what was done in each step. **Use abbreviations and short phrases**, rather than complete sentences, so that you can keep up with the explanation as you write.
   - **Write step-by-step instructions** for each process.
   - **Keep handouts and tests in your notebook,** using either a spiral notebook with pockets or a loose-leaf notebook.
2. **Read the textbook.** Reading a mathematics textbook is different from reading other textbooks. Some suggestions follow.
   - **Skim the material** to get a general idea of the major topics. As you skim the material, circle any words that you do not understand. Read the Chapter Summary and look at the Exercises at the end of each section.
   - **Make note cards** for terms, symbols, and formulas. Review these note cards often to retain the information.
   - **Read for explanation and study the steps to work a problem.** It is essential that you learn *how* to work a problem and *why* the process works rather than memorizing sample problems.
   - **Study any illustrations and other aids,** provided to help you understand a sample problem; then cover up the solution and try to work the problem on your own.
   - **Practice the process.** The more problems you do, the more confident you will become in your ability to do math and perform on tests. When doing your homework, don't give in to frustration. Put your homework aside for a while and come back to it later.
   - **Recite and review.** You should know and understand the example problems in your text well enough to be able to work similar problems on the test. Make note cards with example problems on one side and solutions on the other side.
3. **Work through the Algebra Toolbox.** The Algebra Toolbox will give you a great review of the skills needed for success in each chapter. Note the Key Objectives listed at the beginning of the Toolbox; reread them once you have completed the Toolbox Exercises.

4. **Practice with Skills Check exercises.** Skills Check exercises provide a way to practice your algebra skills before moving on to more applied problems.

5. **Work the Exercises carefully.** The examples and exercises in this book model ways in which mathematics is used in the world. Look for connections between the examples and what you are learning in your other classes. The examples will help you work through the applied exercises in each section.

6. **Prepare for your exams.**
   - **Make a study schedule.** Begin to study at least three days before the test. You should make a schedule, listing those sections of the book that you will study each day. Schedule a sample test to be taken upon completion of those sections. This sample test should be taken at least two days before the test date so that you have time to work on areas of difficulty.
   - **Rework problems.** You should actively prepare for a test. Do *more* than read your notes and the textbook. Do *more* than look over your homework. Review the note cards prepared from your class notes and text. Actually *rework* problems from each section of your book. Use the Chapter Summary at the end of each chapter to be sure you know and understand the key concepts and formulas. Then get some more practice with the Chapter Skills Checks and Review Exercises. Check your answers!
   - **Get help.** Do not leave questions unanswered. Remember to utilize all resources in getting the help you need. Some resources that you might consider using are your fellow classmates, tutors, MyMathLab, the *Student's Solutions Manual*, math videos, and even your teacher. Do not take the gamble that certain questions will not be on the test!
   - **Make a sample test.** Write a sample test by choosing a variety of problems from each section in the book. Then write the problems for your sample test in a different order than they appear in the book. (*Hint*: If you write each problem, with directions, on a separate index card and mix them up, you will have a good sample test.)
   - **Review and relax the night before the test.** The night before the test is best used *reviewing* the material. This may include working one problem from each section, reworking problems that have given you difficulty, or thinking about procedures you have used.
   - **Practice taking tests online.** Ask your professor if MathXL or MyMathLab is available at your school. Both provide online homework, tutorial, and assessment systems for unlimited practice exercises correlated to your textbook. (An access code is required to use these products.)

7. **Develop better math test-taking skills.**
   - **Do a memory download.** As soon as you receive your test, jot down formulas or rules that you will need but are likely to forget. If you get nervous later and forget this information, you merely have to refer to the memory cues that you have written down.
   - **Skip the difficult questions.** Come back to these later or try to work at least one step for partial credit.
   - **Keep a schedule.** The objective is to get the most points. Don't linger over one question very long.
   - **Review your work.** Check for careless errors and make sure your answers make sense.
   - **Use all the time given.** There are no bonus points for turning your test in early. Use extra time for checking your work.

8. **Have fun!** Look for mathematics all around you. Read the newspaper, look at data on government websites, and observe how professionals use mathematics to do their jobs and communicate information to the world.

We have enjoyed teaching this material to our students and watching their understanding grow. We wish you the very best this semester and in your future studies.

Ronald J. Harshbarger
Lisa S. Yocco

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1

Functions, Graphs, and Models; Linear Functions

With digital TV becoming more affordable by the day, the demand for high-definition home entertainment is growing rapidly, with 115 million Americans having digital TV in 2010. The worldwide total was 517 million in 2010, with 40% of TV households having digital TV. More than 401 million digital TV homes are expected to be added between the end of 2010 and the end of 2015, and by 2015 more than 1 billion sets should be in use worldwide. Cell phone use is also on the rise, not just in the United States but throughout the world. By the middle of 2006, the number of subscribers to cell phone carriers had dramatically increased, and the number of total users had reached 4.6 billion by 2010. If the numbers continue to increase at a steady rate, the number of subscribers is expected to reach into the tens of billions over the next few years. These projections and others are made by collecting real-world data and creating mathematical models. The goal of this chapter and future chapters is to use real data and mathematical models to make predictions and solve meaningful problems.

section	objectives	applications
1.1 Functions and Models	Determine graphs, tables, and equations that represent functions; find domains and ranges; evaluate functions and mathematical models	Body temperature, personal computers, stock market, men in the workforce, public health expenditures
1.2 Graphs of Functions	Graph and evaluate functions with technology; graph mathematical models; align data; graph data points; scale data	Personal savings, cost-benefit, voting, U.S. executions, high school enrollment
1.3 Linear Functions	Identify and graph linear functions; find and interpret intercepts and slopes; find constant rates of change; model revenue, cost, and profit; find marginal revenue, marginal cost, and marginal profit; identify special linear functions	Hispanics in the United States, loan balances, revenue, cost, profit, marginal cost, marginal revenue, marginal profit
1.4 Equations of Lines	Write equations of lines; identify parallel and perpendicular lines; find average rates of change; model approximately linear data	Service call charges, blood alcohol percent, inmate population, hybrid vehicle sales, high school enrollment

Algebra TOOLBOX

KEY OBJECTIVES

- Write sets of numbers using description or elements
- Identify sets of real numbers as being integers, rational numbers, and/or irrational numbers
- Identify the coefficients of terms and constants in algebraic expressions
- Remove parentheses and simplify polynomials
- Express inequalities as intervals and graph inequalities
- Plot points on a coordinate system
- Use subscripts to represent fixed points

The Algebra Toolbox is designed to review prerequisite skills needed for success in each chapter. In this Toolbox, we discuss sets, the real numbers, the coordinate system, algebraic expressions, equations, inequalities, absolute values, and subscripts.

Sets

In this chapter we will use sets to write domains and ranges of functions, and in future chapters we will find solution sets to equations and inequalities. A **set** is a well-defined collection of objects, including but not limited to numbers. In this section, we will discuss sets of real numbers, including natural numbers, integers, and rational numbers, and later in the text we will discuss the set of complex numbers. There are two ways to define a set. One way is by listing the **elements** (or **members**) of the set (usually between braces). For example, we may say that a set A contains 2, 3, 5, and 7 by writing $A = \{2, 3, 5, 7\}$. To say that 5 is an element of the set A, we write $5 \in A$. To indicate that 6 is not an element of the set, we write $6 \notin A$. Domains of functions and solutions to equations are sometimes given in sets with the elements listed.

If all the elements of the set can be listed, the set is said to be a **finite set**. If all elements of a set cannot be listed, the set is called an **infinite set**. To indicate that a set continues with the established pattern, we use three dots. For example, $B = \{1, 2, 3, 4, 5, \ldots, 100\}$ describes the finite set of whole numbers from 1 through 100, and $N = \{1, 2, 3, 4, 5, \ldots\}$ describes the infinite set of all whole numbers beginning with 1. This set is called the **natural numbers**.

Another way to define a set is to give its description. For example, we may write $\{x \mid x \text{ is a math book}\}$ to define the set of math books. This is read as "the set of all x such that x is a math book." $N = \{x \mid x \text{ is a natural number}\}$ defines the set of natural numbers, which was also defined by $N = \{1, 2, 3, 4, 5, \ldots\}$ above.

The set that contains no elements is called the **empty set** and is denoted by $\varnothing$.

EXAMPLE 1 ▶

Write the following sets in two ways.

a. The set A containing the natural numbers less than 7.

b. The set B of natural numbers that are at least 7.

SOLUTION

a. $A = \{1, 2, 3, 4, 5, 6\}, A = \{x \mid x \in N, x < 7\}$

b. $B = \{7, 8, 9, 10, \ldots\}, B = \{x \mid x \in N, x \geq 7\}$

The relations that can exist between two sets follow.

Relations Between Sets

1. Sets X and Y are **equal** if they contain exactly the same elements.
2. Set A is called a **subset** of set B if each element of A is an element of B. This is denoted $A \subseteq B$.
3. If sets C and D have no elements in common, they are called **disjoint**.

EXAMPLE 2 ▶ For the sets $A = \{x \mid x \le 9, x \text{ is a natural number}\}$, $B = \{2, 4, 6\}$, $C = \{3, 5, 8, 10\}$:

a. Which of the sets A, B, and C are subsets of A?

b. Which pairs of sets are disjoint?

c. Are any of these three sets equal?

SOLUTION

a. Every element of B is contained in A. Thus, set B is a subset of A. Because every element of A is contained in A, A is a subset of A.

b. Sets B and C have no elements in common, so they are disjoint.

c. None of these sets have exactly the same elements, so none are equal.

The Real Numbers

Because most of the mathematical applications you will encounter in an applied nontechnical setting use real numbers, the emphasis in this text is the **real number system**.* Real numbers can be rational or irrational. **Rational numbers** include integers, fractions containing only integers (with no 0 in a denominator), and decimals that either terminate or repeat. Some examples of rational numbers are

$$-9, \quad \frac{1}{2}, \quad 0, \quad 12, \quad -\frac{4}{7}, \quad 6.58, \quad -7.\overline{3}$$

Irrational numbers are real numbers that are not rational. Some examples of irrational numbers are π (a number familiar to us from the study of circles), $\sqrt{2}$, $\sqrt[3]{5}$, and $\sqrt[3]{-10}$.

The types of real numbers are described in Table 1.1.

Table 1.1

Types of Real Numbers	Descriptions
Natural numbers	1, 2, 3, 4, . . .
Integers	Natural numbers, zero, and the negatives of the natural numbers: . . . , −3, −2, −1, 0, 1, 2, 3, . . .
Rational numbers	All numbers that can be written in the form $\frac{p}{q}$, where p and q are both integers with $q \neq 0$. Rational numbers can be written as terminating or repeating decimals.
Irrational numbers	All real numbers that are not rational numbers. Irrational numbers cannot be written as terminating or repeating decimals.

We can represent real numbers on a **real number line**. Exactly one real number is associated with each point on the line, and we say there is a one-to-one correspondence between the real numbers and the points on the line. That is, the real number line is a graph of the real numbers (see Figure 1.1).

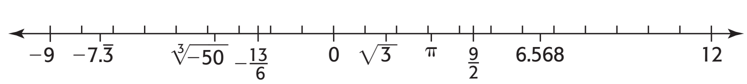


Figure 1.1

* The complex number system will be discussed in the Chapter 3 Toolbox.

Notice the number π on the real number line in Figure 1.1. This special number, which can be approximated by 3.14, results when the circumference of (distance around) any circle is divided by the diameter of the circle. Another special real number is e; it is denoted by

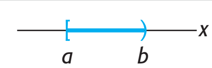
$$e \approx 2.71828$$

We will discuss this number, which is important in financial and biological applications, later in the text.

Inequalities and Intervals on the Number Line

In this chapter, we will sometimes use inequalities and interval notation to describe domains and ranges of functions. An **inequality** is a statement that one quantity is greater (or less) than another quantity. We say that a is less than b (written $a < b$) if the point representing a is to the left of the point representing b on the real number line. We may indicate that the number a is greater than or equal to b by writing $a \geq b$. The subset of real numbers x that lie between a and b (excluding a and b) can be denoted by the **double inequality** $a < x < b$ or by the **open interval** (a, b). This is called an open interval because neither of the endpoints is included in the interval. The **closed interval** $[a, b]$ represents the set of all real numbers satisfying $a \leq x \leq b$. Intervals containing one endpoint, such as $[a, b)$ or $(a, b]$, are called **half-open intervals**. We can represent the inequality $x \geq a$ by the interval $[a, \infty)$, and we can represent the inequality $x < a$ by the interval $(-\infty, a)$. Note that ∞ and $-\infty$ are not numbers, but ∞ is used in $[a, \infty)$ to represent the fact that x increases without bound and $-\infty$ is used in $(-\infty, a)$ to indicate that x decreases without bound. Table 1.2 shows the graphs of different types of intervals.

Table 1.2

Interval Notation	Inequality Notation	Verbal Description	Number Line Graph
(a, ∞)	$x > a$	x is greater than a	
$[a, \infty)$	$x \geq a$	x is greater than or equal to a	
$(-\infty, b)$	$x < b$	x is less than b	
$(-\infty, b]$	$x \leq b$	x is less than or equal to b	
(a, b)	$a < x < b$	x is between a and b, not including either a or b	
$[a, b)$	$a \leq x < b$	x is between a and b, including a but not including b	
$(a, b]$	$a < x \leq b$	x is between a and b, not including a but including b	
$[a, b]$	$a \leq x \leq b$	x is between a and b, including both a and b	

Note that open circles may be used instead of parentheses and solid circles may be used instead of brackets in the number line graphs.

EXAMPLE 3 ▶ Intervals

Write the interval corresponding to each of the inequalities in parts (a)–(e), and then graph the inequality.

a. $-1 \leq x \leq 2$ **b.** $2 < x < 4$ **c.** $-2 < x \leq 3$ **d.** $x \geq 3$ **e.** $x < 5$

SOLUTION

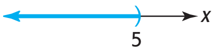
a. $[-1, 2]$

b. $(2, 4)$

c. $(-2, 3]$

d. $[3, \infty)$

e. $(-\infty, 5)$



Algebraic Expressions

In algebra we deal with a combination of real numbers and letters. Generally, the letters are symbols used to represent unknown quantities or fixed but unspecified constants. Letters representing unknown quantities are usually called **variables**, and letters representing fixed but unspecified numbers are called **literal constants**. An expression created by performing additions, subtractions, or other arithmetic operations with one or more real numbers and variables is called an **algebraic expression**. Unless otherwise specified, the variables represent real numbers for which the algebraic expression is a real number. Examples of algebraic expressions include

$$5x - 2y, \quad \frac{3x - 5}{12 + 5y}, \quad \text{and} \quad 7z + 2$$

A **term** of an algebraic expression is the product of one or more variables and a real number; the real number is called a **numerical coefficient** or simply a **coefficient**. A constant is also considered a term of an algebraic expression and is called a **constant term**. For instance, the term $5yz$ is the product of the factors 5, y, and z; this term has coefficient 5.

Polynomials

An algebraic expression containing a finite number of additions, subtractions, and multiplications of constants and nonnegative integer powers of variables is called a **polynomial**. When simplified, a polynomial cannot contain negative powers of variables, fractional powers of variables, variables in a denominator, or variables inside a radical. The expressions $5x - 2y$ and $7z^3 + 2y$ are polynomials, but $\dfrac{3x - 5}{12 + 5y}$ and $3x^2 - 6\sqrt{x}$ are not polynomials. If the only variable in the polynomial is x, then the polynomial is called a **polynomial in *x***. The general form of a polynomial in x is

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where a_0 and each coefficient $a_n, a_{n-1}, \ldots$ are real numbers and each exponent $n, n - 1, \ldots$ is a positive integer.

For a polynomial in the single variable x, the power of x in each term is the **degree** of that term, with the degree of a constant term equal to 0. The term that has the highest power of x is called the **leading term** of the polynomial, the coefficient of this term is the **leading coefficient**, and the degree of this term is the **degree of the polynomial**. Thus, $5x^4 + 3x^2 - 6$ is a fourth-degree polynomial with leading coefficient 5. Polynomials with one term are called **monomials**, those with two terms are called **binomials**, and those with three terms are called **trinomials**. The right side of the equation $y = 4x + 3$ is a first-degree binomial, and the right side of $y = 6x^2 - 5x + 2$ is a second-degree trinomial.

EXAMPLE 4 ▶

For each polynomial, state the constant term, the leading coefficient, and the degree of the polynomial.

a. $5x^2 - 8x + 2x^4 - 3$ **b.** $5x^2 - 6x^3 + 3x^6 + 7$

SOLUTION

a. The constant term is -3; the term of highest degree is $2x^4$, so the leading coefficient is 2 and the degree of the polynomial is 4.

b. The constant term is 7; the term of highest degree is $3x^6$, so the leading coefficient is 3 and the degree of the polynomial is 6.

Terms that contain exactly the same variables with exactly the same exponents are called **like terms**. For example, $3x^2y$ and $7x^2y$ are like terms, but $3x^2y$ and $3xy$ are not. We can *simplify* an expression by adding or subtracting the coefficients of the like terms. For example, the simplified form of

$$3x + 4y - 8x + 2y \quad \text{is} \quad -5x + 6y$$

and the simplified form of

$$3x^2y + 7xy^2 + 6x^2y - 4xy^2 - 5xy \quad \text{is} \quad 9x^2y + 3xy^2 - 5xy$$

Removing Parentheses

We often need to remove parentheses when simplifying algebraic expressions and when solving equations. Removing parentheses frequently requires use of the **distributive property**, which says that for real numbers a, b, and c, $a(b + c) = ab + ac$. Care must be taken to avoid mistakes with signs when using the distributive property. Multiplying a sum in parentheses by a negative number changes the sign of each term in the parentheses. For example,

$$-3(x - 2y) = -3(x) + (-3)(-2y) = -3x + 6y$$

and

$$-(3xy - 5x^3) = -3xy + 5x^3$$

We add or subtract (**combine**) algebraic expressions by combining the like terms. For example, the sum of the expressions $5sx - 2y + 7z^3$ and $2y + 5sx - 4z^3$ is

$$\begin{aligned}(5sx - 2y + 7z^3) + (2y + 5sx - 4z^3) &= 5sx - 2y + 7z^3 + 2y + 5sx - 4z^3 \\ &= 10sx + 3z^3\end{aligned}$$

and the difference of these two expressions is

$$\begin{aligned}(5sx - 2y + 7z^3) - (2y + 5sx - 4z^3) &= 5sx - 2y + 7z^3 - 2y - 5sx + 4z^3 \\ &= -4y + 11z^3\end{aligned}$$

The Coordinate System

Much of our work in algebra involves graphing. To graph in two dimensions, we use a rectangular coordinate system, or **Cartesian coordinate system**. Such a system allows us to assign a unique point in a plane to each ordered pair of real numbers. We construct the coordinate system by drawing a horizontal number line and a vertical number line so that they intersect at their origins (Figure 1.2). The point of intersection is called the **origin** of the system, the number lines are called the coordinate **axes**, and the plane is divided into four parts called **quadrants**. In Figure 1.3, we call the horizontal axis the ***x*-axis** and the vertical axis the ***y*-axis**, and we denote any point in the plane as the ordered pair (x, y).

The ordered pair (a, b) represents the point P that is $|a|$ units from the y-axis (right if a is positive, left if a is negative) and $|b|$ units from the x-axis (up if b is positive, down if b is negative). The values of a and b are called the **rectangular coordinates** of the point. Figure 1.3 shows point P with coordinates (a, b). The point is in the second quadrant, where $a < 0$ and $b > 0$.

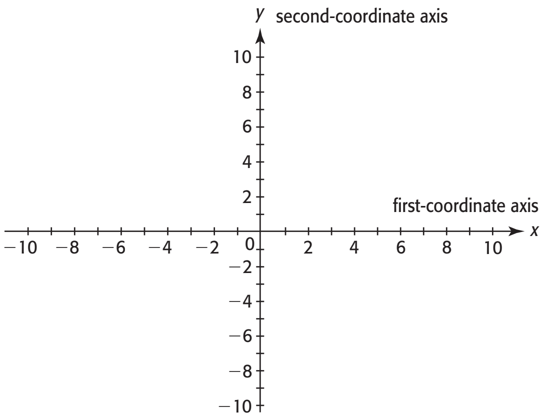


Figure 1.2

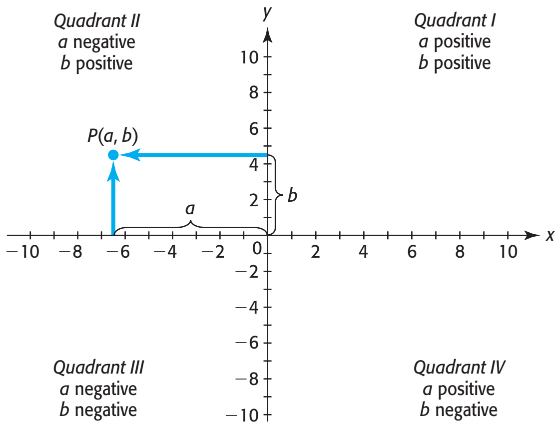


Figure 1.3

Subscripts

We sometimes need to distinguish between two y-values and/or x-values in the same problem or on the same graph, or to designate literal constants. It is often convenient to do this by using **subscripts**. For example, if we have two fixed but unidentified points on a graph, we can represent one point as (x_1, y_1) and the other as (x_2, y_2). Subscripts also can be used to designate different equations entered in graphing utilities; for example, $y = 2x - 5$ may appear as $y_1 = 2x - 5$ when entered in the equation editor of a graphing calculator.

Toolbox EXERCISES

1. Write "the set of all natural numbers N less than 9" in two different ways.

2. Is it true that $3 \in \{1, 3, 4, 6, 8, 9, 10\}$?

3. Is A a subset of B if $A = \{2, 3, 5, 7, 8, 9, 10\}$ and $B = \{3, 5, 8, 9\}$?

4. Is it true that $\frac{1}{2} \in N$ if N is the set of natural numbers?

5. Is the set of integers a subset of the set of rational numbers?

6. Are sets of rational numbers and irrational numbers disjoint sets?

Identify the sets of numbers in Exercises 7–9 as containing one or more of the following: integers, rational numbers, and/or irrational numbers.

7. $\{5, 2, 5, 8, -6\}$

8. $\left\{\frac{1}{2}, -4.1, \frac{5}{3}, 1\frac{2}{3}\right\}$

9. $\left\{\sqrt{3}, \pi, \frac{\sqrt[3]{2}}{4}, \sqrt{5}\right\}$

In Exercises 10–12, express each interval or graph as an inequality.

10. 

11. $[-3, 3]$

12. $(-\infty, 3]$

In Exercises 13–15, express each inequality or graph in interval notation.

13. $x \le 7$

14. $3 < x \le 7$

15. 

In Exercises 16–18, graph the inequality or interval on a real number line.

16. $(-2, \infty)$

17. $5 > x \ge 2$

18. $x < 3$

In Exercises 19–21, plot the points on a coordinate system.

19. $(-1, 3)$

20. $(4, -2)$

21. $(-4, 3)$

22. Plot the points $(-1, 2)$, $(3, -1)$, $(4, 2)$, and $(-2, -3)$ on the same coordinate system.

23. Plot the points (x_1, y_1) and (x_2, y_2) on a coordinate system if

$$x_1 = 2, y_1 = -1, x_2 = -3, y_2 = -5$$

Determine if each expression in Exercises 24–27 is a polynomial. If it is, state the degree of the polynomial.

24. $14x^4 - 6x^3 + 9x - 7$

25. $\dfrac{5x - 8}{3x + 2}$

26. $10x - \sqrt{y}$

27. $-12x^4 + 5x^6$

For each algebraic expression in Exercises 28 and 29, give the coefficient of each term and give the constant term.

28. $-3x^2 - 4x + 8$

29. $5x^4 + 7x^3 - 3$

30. Find the sum of $z^4 - 15z^2 + 20z - 6$ and $2z^4 + 4z^3 - 12z^2 - 5$.

31. Simplify the expression

$$3x + 2y^4 - 2x^3y^4 - 119 - 5x - 3y^2 + 5y^4 + 110$$

Remove the parentheses and simplify in Exercises 32–37.

32. $4(p + d)$

33. $-2(3x - 7y)$

34. $-a(b + 8c)$

35. $4(x - y) - (3x + 2y)$

36. $4(2x - y) + 4xy - 5(y - xy) - (2x - 4y)$

37. $2x(4yz - 4) - (5xyz - 3x)$

1.1 Functions and Models

KEY OBJECTIVES

- Determine if a table, graph, or equation defines a function
- Find the domains and ranges of functions
- Create a scatter plot of a set of ordered pairs
- Use function notation to evaluate functions
- Apply real-world information using a mathematical model

SECTION PREVIEW Body Temperatures

One indication of illness in children is elevated body temperature. There are two common measures of temperature, Fahrenheit (°F) and Celsius (°C), with temperature measured in Fahrenheit degrees in the United States and in Celsius degrees in many other countries of the world. Suppose we know that a child's normal body temperature is 98.6°F and that his or her temperature is now 37°C. Does this indicate that the child is ill? To help decide this, we could find the Fahrenheit temperature that corresponds to 37°C. We could do this easily if we knew the relationship between Fahrenheit and Celsius temperature scales. In this section, we will see that the relationship between these measurements can be defined by a **function** and that functions can be defined numerically, graphically, verbally, or by an equation. We also explore how this and other functions can be applied to help solve problems that occur in real-world situations. ■

Function Definitions

There are several techniques to show how Fahrenheit degree measurements are related to Celsius degree measurements.

One way to show the relationship between Celsius and Fahrenheit degree measurements is by listing some Celsius measurements and the corresponding Fahrenheit measurements. These measurements, and any other real-world information collected in numerical form, are called **data**. These temperature measurements can be shown in a table (Table 1.3).

Table 1.3

Celsius Degrees (°C)	−20	−10	−5	0	25	50	100
Fahrenheit Degrees (°F)	−4	14	23	32	77	122	212

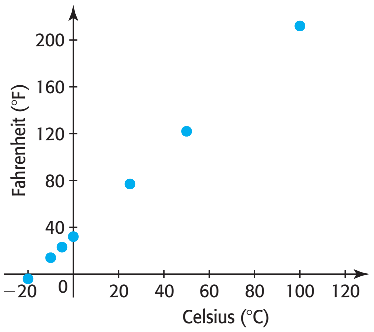


Figure 1.4

This relationship is also defined by the set of ordered pairs

$$\{(-20, -4), (-10, 14), (-5, 23), (0, 32), (25, 77), (50, 122), (100, 212)\}$$

We can picture the relationship between the measurements with a graph. Figure 1.4 shows a **scatter plot** of the data—that is, a graph of the ordered pairs as points. Table 1.3, the set of ordered pairs below the table, and the graph in Figure 1.4 define a **function** with a set of Celsius temperature **inputs** (called the **domain** of the function) and a set of corresponding Fahrenheit **outputs** (called the **range** of the function). A function that will give the Fahrenheit temperature measurement F that corresponds to *any* Celsius temperature measurement C between −20°C and 100°C is the equation

$$F = \frac{9}{5}C + 32$$

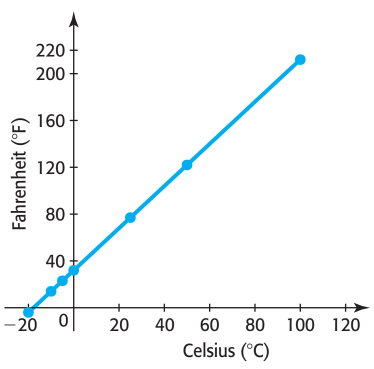


Figure 1.5

This equation defines F as a function of C because each input C results in exactly one output F. Its graph, shown in Figure 1.5, is a line that contains the points on the scatter plot in Figure 1.4 as well as other points. If we consider only Celsius temperatures from −20 to 100, then the domain of this function defined by the equation above is $-20 \le C \le 100$ and the resulting range is $-4 \le F \le 212$.